Flow Behavior of Polyacrylamide Solution. III. Mathematical Treatment

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ABSTRACT: The flow behavior of polyacrylamide solutions was systematically determined over a wide range of temperatures (20–50°C) and concentrations (20–50 ppm) by using a coaxial cylinder viscometer. The results indicated that the rheological behavior of low-concentration polyacrylamide solution behaves similar to non-Newtonian fluids at all these concentrations. The effect of temperature on the consistency coefficient and flow behavior index of polyacrylamide solution of the different concentrations followed an Arrhenius-type relationship. Moreover, the effect of concentration on consistency coefficient and flow behavior index followed an exponential-law relationship at the temperatures used. The rheological constants for the Arrhenius and exponential-law models were determined. The combined effect of temperature and concentration on the coefficient of dynamic shear stress can be represented by a single equation: shear stress $= 2.446 \times 10^{-7}\exp(0.0639C + 3613/\mathrm{RT})(\text{shear rate})^{2.337\exp(-0.00707C-245/\mathrm{RT})}$. © 2001 John Wiley & Sons, Inc. J Appl Polym Sci 82: 2784–2789, 2001

Key words: polyacrylamide; flow behavior

INTRODUCTION

The polyacrylamide solution is important for using coagulation of wastewater treatment process. Hence, the rheological behavior of polyacrylamide solution is very important in solution properties and engineering calculations related to the handling of the polymer solution for addition in wastewater treatment.[1–4] The effect of various components of polyacrylamide solution on its flow behavior under low shear rate was recently reported.[5] In the case of polyacrylamide solution, the rheological behavior of polyacrylamide homopolymer solution in water depends not only on shear rate, but also on variables such as concentration and temperature. The effect of various medium-concentration polyacrylamide solutions on its flow behavior was recently reported.[6] It is unknown whether the flow behavior of polyacrylamide solution conforms to power law for a wide range of shear rates. The power law is given as

$$\tau = K\dot{\gamma}^{n} \tag{1}$$

where τ is the shear stress (N/m^2); K is the consistency coefficient (N s^n/m^2); $\dot{\gamma}$ is the shear rate (s^{-1}); and n is flow behavior index (dimensionless). These constants can be used as an estimate of the polymer solution viscosity when there are no experimentally determined values.

The purposes of this work was to devise a power-law model for the flow behavior of polyacrylamide solution at various temperatures and concentrations, to evaluate the effect of various factors on

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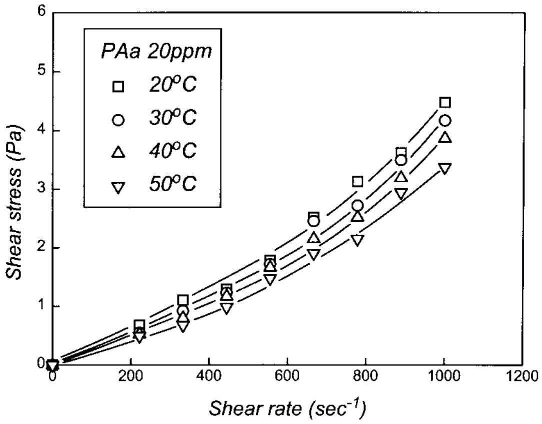

Figure 1 The relationship between shear stress and shear rate of the polyacrylamide solution rheograms at various temperatures.

the flow pattern, and finally, to determine the applicability of this mathematical model.

EXPERIMENTAL

Polyacrylamide (EP grade) used in these experiments was obtained from Acros Organics (USA). The materials and preparation of the polyacrylamide solution's experimental procedure were the same as those employed in the previous article.[7] The rheological measurements were carried out by using a Rotovisco RV 12 (Haake) concentric cylinder viscometer equipped with an M-500-type measurement attachment, which can transmit a maximum toque of 4.90 N-cm by using NV-type pair coaxial cylinders. A thermostatic bath controls the working temperature within the range of 20–50 ± 0.1°C. Rotor speeds were variable in the range of 0–999 rpm, which enabled rheograms (shear stress versus shear rate diagram) to be constructed.

RESULTS AND DISCUSSION

Rheological Behavior of Polyacrylamide Concentration

The rheological behavior of low-concentration polyacrylamide solutions was studied in the concentration range of 20–50 ppm by weight and in the temperature range of 20–50°C. Figure 1 shows the experimental results of shear stress (τ) versus shear rate ($\dot{\gamma}$), which indicate the non-Newtonian behavior of these solutions. Figure 1 shows the experimental results obtained for the polyacrylamide solution at 20 ppm at four different temperatures, the shear stress increased with increasing the shear rate. Similar levels were observed at other concentrations also, for example, 30, 40, and 50 ppm. It can be seen from Figure 1 that at a higher temperature, shear stress decreased, and at a higher concentration polyacrylamide solution, shear stress increased. Hence, the polyacrylamide solution studied in this article behaved similarly to a non-Newtonian fluid. The values of the consistency coefficient and flow behavior index were obtained using eq. (1) by fitting the experimental results to the non-Newtonian relationship. Figure 2 shows the experimental results obtained for the polyacrylamide solution at 30°C at four different concentrations, 20, 30, 40, and 50 ppm. The shear stress increased with increasing the shear rate. Similar levels were observed at other temperatures, 30, 40, and 50°C. It can be seen from Figure 2 that at a higher concentration polyacrylamide, shear stress increased, and at a higher temperature, shear stress decreased.

Using logarithms, eq. (1) becomes

$$\ln(\tau) = \ln(K) + n \ln(\dot{\gamma}) \tag{2}$$

The $\ln(\dot{\gamma})$ dependence of $\ln(\tau)$ for 30 ppm at various temperatures is given in Figure 3. According to eq. (2), the $\ln(K)$ and n were evaluated by linear regression analysis of the points and are shown in Figure 3.

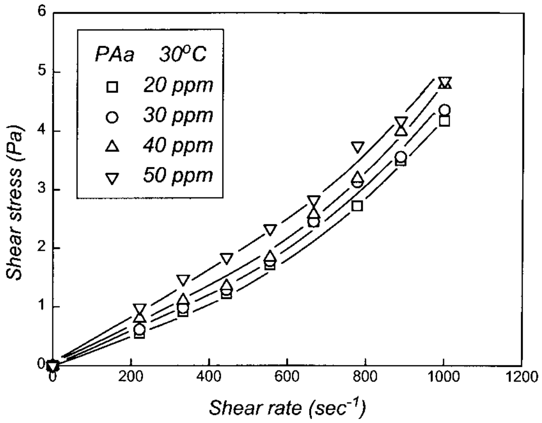

Figure 2 The relationship between shear stress and shear rate of the polyacrylamide solution rheograms at various concentrations.

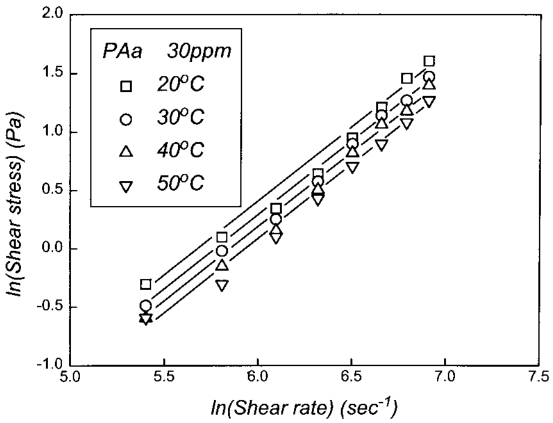

Figure 3 Logarithmic plots of shear stress and shear rate of the polyacrylamide solution rheograms at various temperatures.

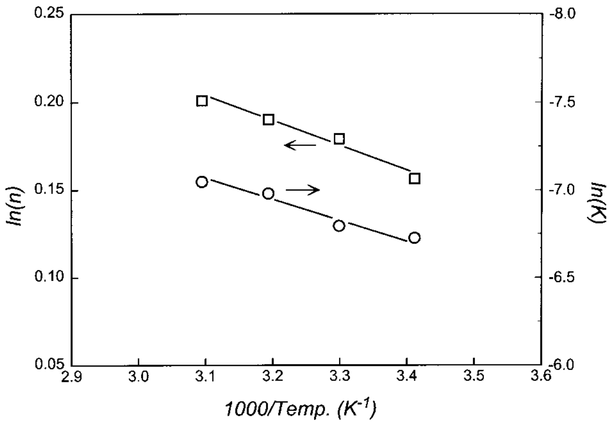

Figure 5 Logarithmic plots of rheological behavior constants and temperature of the polyacrylamide solution rheograms.

Effect of Temperature on the Rheological Characteristics

The relationship between the temperature and rheological characteristics (i.e., consistency coefficient and flow behavior index) is expressed by the following Arrhenius relationship, shown in Figure 4:

$$n = n_0 \exp(-Ea/RT) \tag{3}$$

$$K = K_0 \exp(Ea/RT) \tag{4}$$

where n_0 and K_0 represent constants; Ea represents the activation energy (cal/g-mol); R represents the gas constant (cal/g-mol K); and T is the absolute temperature, K. As shown in Figure 4, an increase in the temperature leads to a decrease in the consistency coefficient and an increase in the flow behavior index.

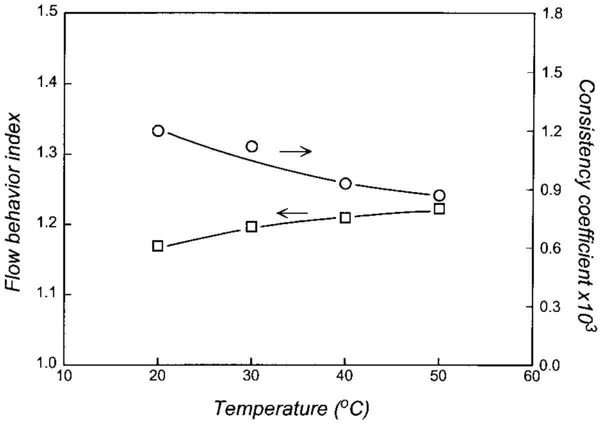

Figure 4 The effect of temperature on the rheological behavior constants of polyacrylamide solution rheograms.

The temperature dependence of the ln(n) and ln(K) for different polyacrylamide concentration solutions is given in Figure 5. An Arrhenius-type dependence of ln(n) and ln(K) on temperature in the temperature range 20–50°C is observed. From this dependence, the activation energies were evaluated by linear regression analysis of the data points and are shown in Figure 5. It was found that the energy is 0.245 and 3.61 kcal/g-mol and is independent of the concentration of polyacrylamide solution in the range of 20–50 ppm.

Effect of the Concentration on the Rheological Characteristics

The relationship between the polyacrylamide concentration and rheological characteristics (i.e., consistency coefficient and flow behavior index) is expressed by different expressions, generally of the power-law type or the exponential-law type$^{8-10}$:

$$A = A_1(C)^{a_1} \quad \text{where } A = n, K \tag{5}$$

$$B = B_2 \exp(a_2 C) \quad \text{where } B = n, K \tag{6}$$

In both equations, n_i and K_i are flow behavior index and consistency coefficient, respectively, a_i and b_i are constants, and C is the concentration in ppm. To calculate the different parameters of these equations, the ln(C) dependence of the ln(n)

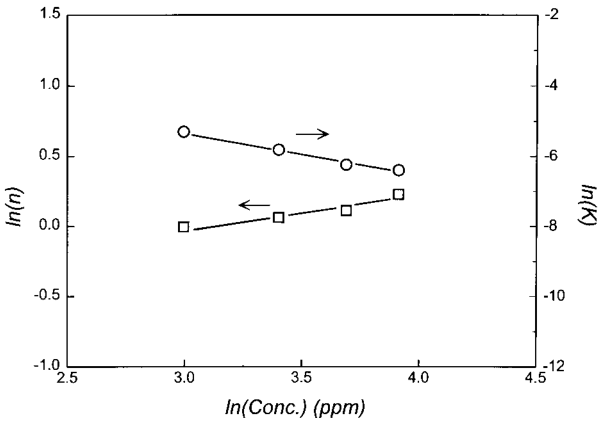

Figure 6 Logarithmic plots of rheological behavior constants and concentration of the polyacrylamide solution rheograms.

and ln(K) for different temperatures using eq. (5) is given in Figure 6. According to eq. (5), a straight line should result from ln(n), ln(K), and ln(C). The dependence ln(C) for different temperatures is given in Figure 6 and illustrates that for those cases a straight line exists between the ln(n), ln(K) of the polyacrylamide solution and the ln(C), where slopes are a_1 and b_1, respectively. From this dependence, a_1 and b_1 were evaluated by linear regression analysis. These fits were obtained by least-squares method, showing in both cases that the fit and the estimates of the parameters were significant at a probability level of 95%.

The concentration dependence of the ln(n) and ln(K) for different temperatures using eq. (6) is given in Figure 7. According to eq. (6), a straight line should result from ln(n), ln(K), and the concentration. The dependence of concentration on different temperature is given in Figure 7 and illustrates that for those cases a straight line exists between the ln(n) and ln(K) of the polyacrylamide solution and the concentration where slopes are a_2 and b_2, respectively. From this dependence, a_2 and b_2 were evaluated by linear regression analysis. These fits were obtained by least-squares method, showing in both cases that the fit and estimates of the parameters were significant at a probability level of 96%.

From the values of the regression coefficient obtained, the experimental results of flow behavior and polyacrylamide concentration were fitted to the linear form of eqs. (3) and (4) by the least-squares method to obtain the estimates of the parameters of the model. The best fit was found when the exponential relation [eqs. (5) and (6)] was used for describing the effect on the rheological behavior index and consistency coefficient of the polyacrylamide solution.

Combined Effect of Temperature and Concentration on the Rheological Behavior

For practical engineering applications, it is useful to get a combined equation describing the combined effect of temperature and concentration on polyacrylamide solution rheological behavior.

From the results obtained in the earlier section, the following equations were proposed:

$$\tau = K\dot{\gamma}^n$$

$$A = A_3\exp(a_3C - Ea/RT) \quad \text{where } A = n, K \quad (7)$$

$$B = B_4C^{a_4}\exp(-Ea/RT) \quad \text{where } B = n, K \quad (8)$$

The rheological characteristics were fitted to these equations by multiple linear regressions calculated by Box[11] and a computer program was used to calculate the linear regression. The values of the constants obtained are given in Table I. The fit and the estimates of the constants are significant at a probability level of 98%. From the results obtained, it seems that eq. (7) best describes the combined effect of temperature and concentration. Therefore, for the range of concentrations and temperatures used, a combined equation is

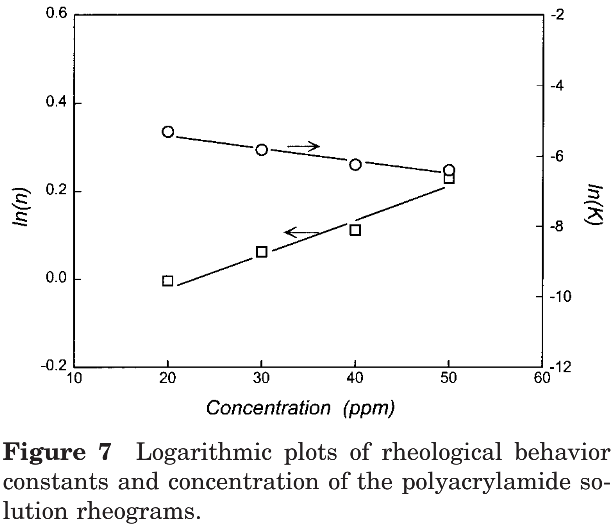

Figure 7 Logarithmic plots of rheological behavior constants and concentration of the polyacrylamide solution rheograms.

Table I The Relationship Between Shear Stress and Concentration at Various Temperatures

	$\tau = K\dot{\gamma}^n$	
	Power Model	Exponential Model
n Value	$n = n_4(C)^{a_4}\exp(-Ea/RT)$	$n = n_3\exp(a_3C - Ea/RT)$
K Value	$K = K_4(C)^{b_4}\exp(Ea'/RT)$	$K = K_3\exp(b_3C + Ea'/RT)$
n_i	3.917	2.337
a_i	−0.02182	−0.00707
Ea (cal/mole)	245	245
K_i	2.222×10^{-9}	2.446×10^{-7}
b_i	1.983	0.0639
Ea' (cal/mol)	3613	3613
Corr. coeff.	0.984	0.985

proposed to describe the rheological characteristics of low concentration of polyacrylamide solution,

$$\tau = K\dot{\gamma}^n \tag{9}$$

where $n = 2.337\ \exp(-0.00707C - 245/RT)$, $K = 2.446 \times 10^{-7}\ \exp(0.0639C + 3613/RT)$, in which C represents the concentration, ppm; and T represents the temperature in K.

A test was made on the applicability of eq. (9). This was checked by plotting experimental τ values versus calculated τ values by using eq. (9). This type of plot is shown in Figure 8, and it can be seen that for the most part remarkably straight lines are formed. This results, in the combined effect of temperature and concentration, suggesting that eq. (9) is reasonable to show the rheological behavior of low-concentration polyacrylamide solution.

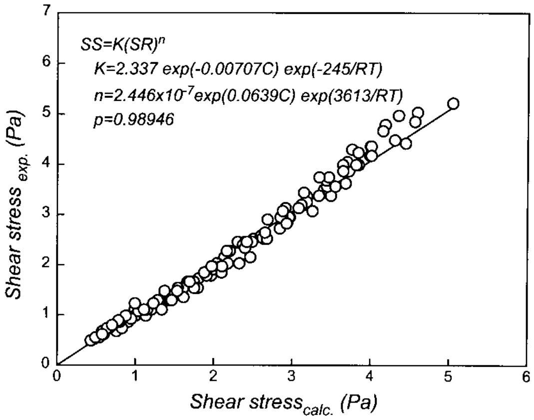

Figure 8 The relationship between the experimental shear stress and calculated shear stress.

CONCLUSION

The effect of temperature and concentration on rheological behavior of polyacrylamide solution was examined in the temperature range of 20–50°C and the concentration range of 20–50 ppm, using a coaxial cylinder viscometer. The rheological behavior of low concentration polyacrylamide solution acts as non-Newtonian fluids. It was found that the shear stress of low concentration of polyacrylamide solution decreased with increasing temperature and increased with increasing concentration. The consistency coefficient of polyacrylamide solution decreased and flow behavior index of polyacrylamide solution increased with an increase in temperature. It was found that the effect of temperature on the consistency coefficient and flow behavior index of polyacrylamide solution of the different concentrations followed an Arrhenius-type relationship. Moreover, the consistency coefficient of polyacrylamide solution increased and flow behavior index of polyacrylamide solution decreased with increasing polyacrylamide concentration. The power law and exponential law were used to fit the experimental data. The rheological constants for the exponential law and power law models were determined. The empirical equation $\tau = K\gamma$, where $K = 2.446 \times 10^{-7}\exp(0.0639C + 3613/RT)$ and $n = 2.337\ \exp(-0.00707C - 245/RT)$, was given for describing the combined effect of temperature and concentration on the behavior properties of low concentration of polyacrylamide solution. It was found that the experimental data and the equation correlate well with each other.

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